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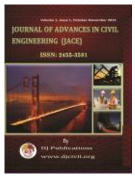

RESEARCH ARTICLE

Mathematical Modelling & Simulation of Mixing of Salt in 3-Interconnected Tanks

Jamshaid ul Rahman[1], *Muhammad Raheel Mohyuddin[1, 2], Syed Zahoor[1], Naveed Anjum[3]
[1]Department of Mathematics & Statistics, Caledonian College of Engineering, Oman.
[2]Department of Mathematics, NCBAE, Gujrat, Pakistan.
[3]Department of Mathematics, GC University Faisalabad, Pakistan

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ABSTRACT

This paper models and simulates the interconnected water tanks that are useful in industrial control systems specifically in modelling of fluid dynamical systems appearing in instrumentation engineering. These models are based on simultaneous ordinary differential equations that play a vital role in instrumental industrial systems. Elementary control principle of three connected water tanks system is to maintain a constant level of water in these tanks that have different paths for inflow and outflow of water. This paper gives the mathematical modeling of a three tanks system and a procedure for generating a simulation model in MATLAB /Simulink that is helpful for the simulation & mathematical modeling of many other scientific instruments.

Keywords: Modelling, 3-Tank system, MATLAB, Simulink, Differential equations.

1. INTRODUCTION

Tank measures the level and volume of the fillings of fresh water, cargo, fuel and oil. Controlling the level in multiple interconnected tanks is very common in water purification, petroleum and food industries. The modelling of these has been given mathematically, in [1, 2]. Modelling based on the water tank system is mathematically given by the ordinary differential equation in [3]. Some impressive numerical simulation is given in [5]. Using these mathematical equations the simulink modelling of interconnected tanks is given in [3, 6].

The Silvretta Reservoir is the most impressive and highest artificial lake in Europe [7]. Its model is the largest and highest coupled tank system in the world. A large number of fluid instruments contain such type of mathematical equations like DTS200 based on the same simultaneous linear ordinary differential equations [4]. The modelling and simulation of ordinary differential equations can be simulated by using simulink [8].

Keeping these references in front we are going to construct a mathematical and simulink Model for 3- interconnected tanks.

2. MATHEMATICAL MODELING

A mathematical model is the description of a system using mathematical thoughts, concepts, ideas and notions [4]. This derivation is based on properties of individual components and on mathematical modelling of single and quadruple tank system [1]. If a tank contains M (litre) of water that dissolves Q (kg) of salt & P (litres) of brian containing S kg of dissolved salt enters into the tank per minute [3] then we have

in-flow of salt = P(litres)×S (kg of salt/litre)
One litre in tank = A(t)/M of salt, then
P litres outgoing will be = P × A(t)/M

Therefore, balance of flow rates is:

Rate of Change of salt in tank = inflow rate – outflow rate and is shown in equation (2.1) and (2.2).

$$\frac{dA(t)}{dt} = P \times S - \frac{P}{M}A(t) \qquad (2.1)$$

$$\frac{dA(t)}{dt} + \frac{P}{M}A(t) = P \times S \qquad (2.2)$$

Most of simultaneous linear models consist of 3- ODEs in 3- unknowns having the

*Corresponding author. Tel.: +96899852897
Email address: mraheel.mohyuddin@caledonian.edu.om (M.R.Mohyuddin)
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following forms is shown in equation (2.3), (2.4) and (2.5).

$$\frac{dA_1(t)}{dt} = a_{11}A_1(t) + a_{12}A_2(t) + a_{13}A_3(t) \quad (2.3)$$

$$\frac{dA_2(t)}{dt} = a_{21}A_1(t) + a_{22}A_2(t) + a_{23}A_3(t) \quad (2.4)$$

$$\frac{dA_3(t)}{dt} = a_{31}A_1(t) + a_{32}A_2(t) + a_{33}A_3(t) \quad (2.5)$$

3. FORMULATION

Our problem is based on flow of incompressible fluids and so we have a hydraulic system given in figure 1. There are three interconnected tanks where tank T_1 has 100 litres of water having 200 kg of salt. The liquid enters into the tank at a constant rate of 2 litre/minute and the mixture is kept uniform. The tank T_2 has initially 100 litres of pure water and enters 2 litres per minute to T_2 and T_3 tanks. T_3 has also 100 litres of pure water initially and its flow to tank-2 is 2 litres per minute.

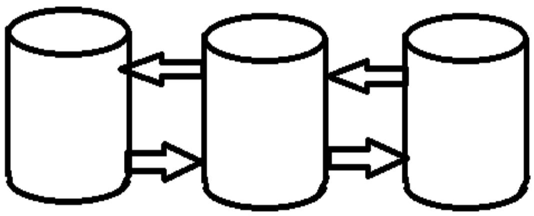

Figure 1.Interconnected tanks

If $A_1(t)$ is the amount of salt in tank-1 and $A_2(t)$ and $A_3(t)$ amounts of salt in tank-2 and tank-3 respectively, then for a single tank the time rate of change of $A_1(t)$ is equal to the inflow minus the outflow:

$$\frac{dA_1(t)}{dt} = 0.02A_2(t) - 0.02A_1(t) \quad (3.1)$$

Similarly for tank 2 and tank 3, we get equation (3.2) and (3.3) respectively

$$\frac{dA_2(t)}{dt} = \frac{inflow}{m} - \frac{outflow}{m} = 0.02A_1(t) + 0.02A_2(t) + 0.02A_3(t) + 0.02A_4(t) \quad (3.2)$$

$$\frac{dA_3(t)}{dt} = \frac{inflow}{m} - \frac{outflow}{m} = 0.02A_2(t) - 0.02A_3(t) \quad (3.3)$$

Writing (3.1) to (3.3) in matrix form

$$\begin{bmatrix} dA_1/dt \\ dA_2/dt \\ dA_3/dt \end{bmatrix} = \begin{bmatrix} -0.02 & 0.02 & 0 \\ 0.02 & -0.04 & 0.02 \\ 0 & 0.02 & -0.02 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

The initial conditions for (3.1) to (3.3) are given by

$$A_1(0) = 200,\ A_2(0) = 0,\ A_3(0) = 0$$

Figure A1 shows the MATLAB Simulink model

4. DISCUSSION

The complete process with time intervals are shown in figures A2 to A5 that give the presences of salt in 3-tanks. It is clear from figure A2 that the salt in tank 1 starts to decrease from 200 to attain a fixed level and after 200 minutes 50 kg of salt remain in it. Tank 2 initially has no salt in it (as shown in figure A3) but after 40 minutes it obtains a maximum level. Similarly, tank 3, contains no salt initially as shown in figure A4 but after 100 minutes it achieves maximum value. Therefore as we increase the time the amount of salt attain a fixed level. The comparison of amounts of salt in three tanks are represented in figure A5.

5. CONCLUSION

In this paper we study modelling of MATLAB simulink Model of 3-Tank systems. This simulink model provides direct simulations of ordinary differential equation based model as compared to numerical techniques. Mathematical modelling for the simulation of this model helps in furnishing a convenient and flexible design that offers performance in mixing of salt in 3-interconnected tanks.

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APPENDIX A

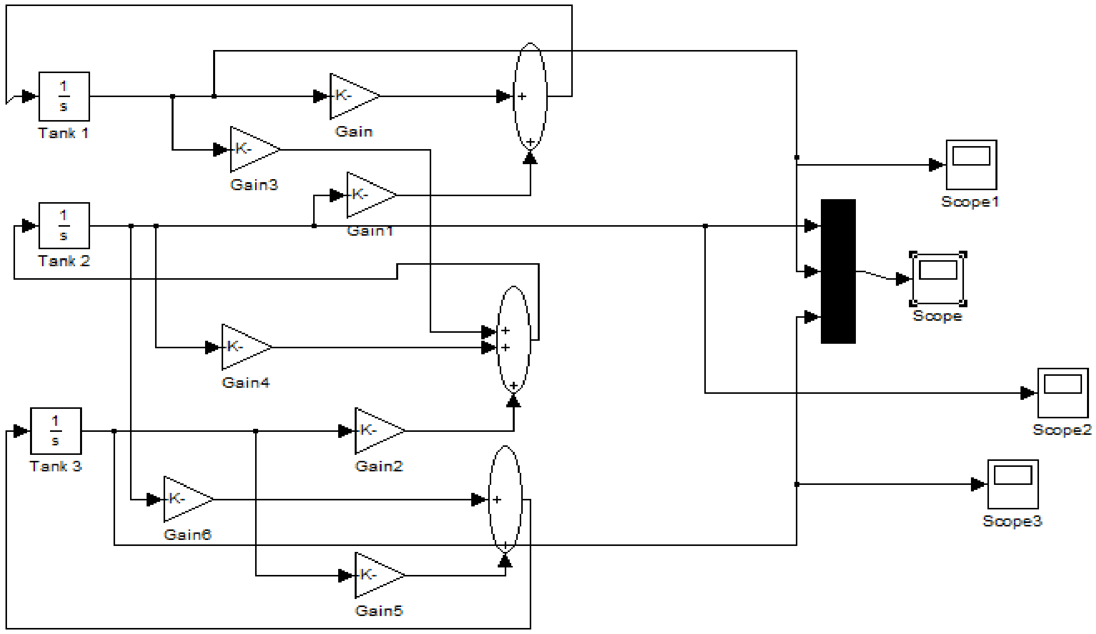

Figure A1.MATLAB Simulink Model

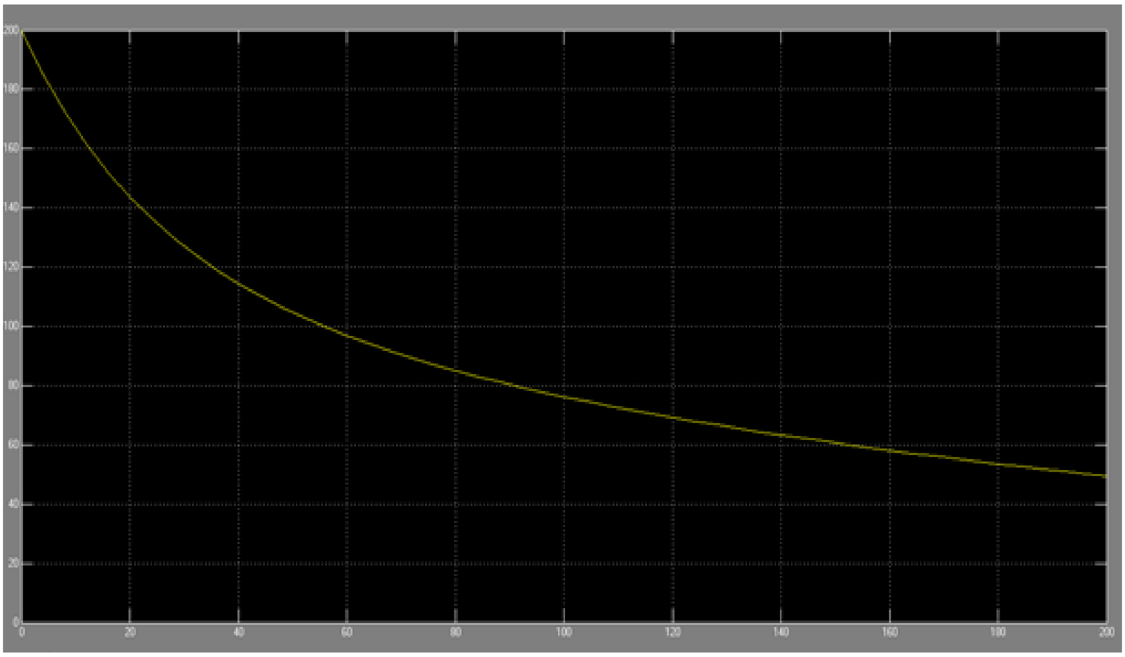

Figure A2.Scope 1

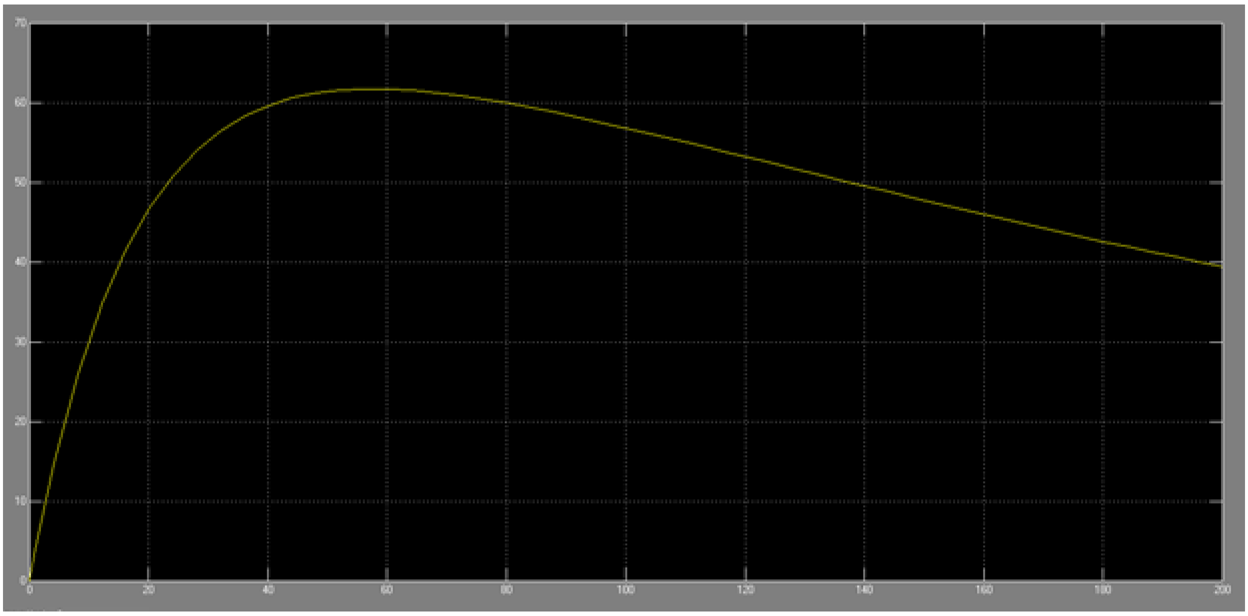

Figure A3.Scope 2

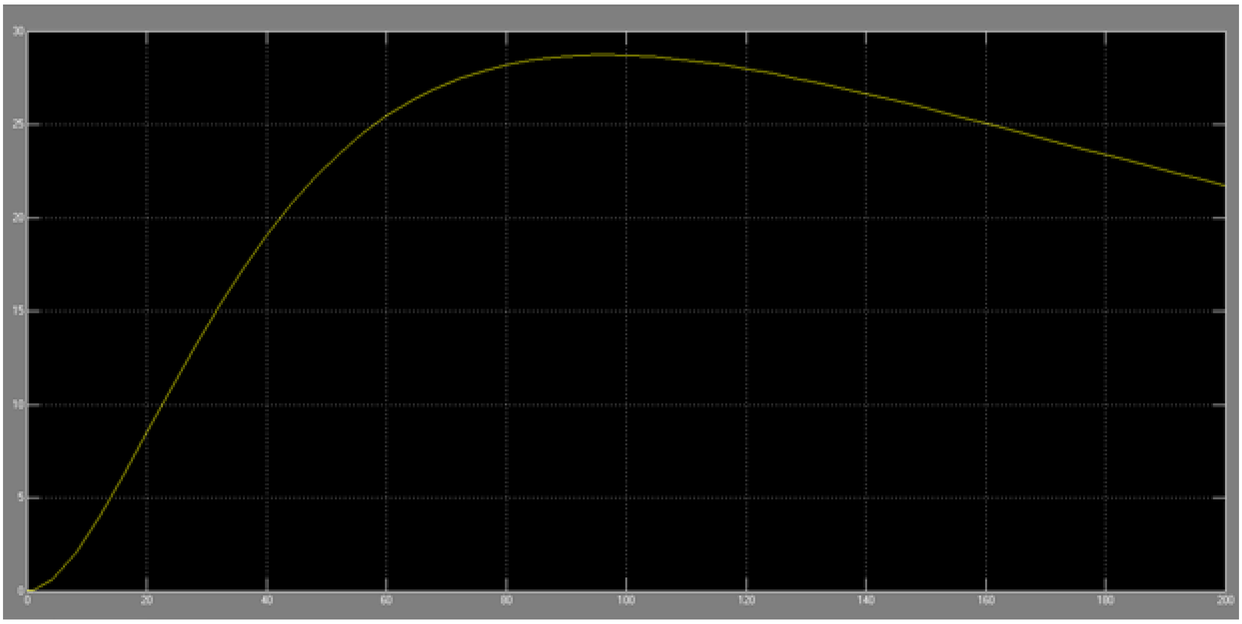

Figure A4.Scope 3

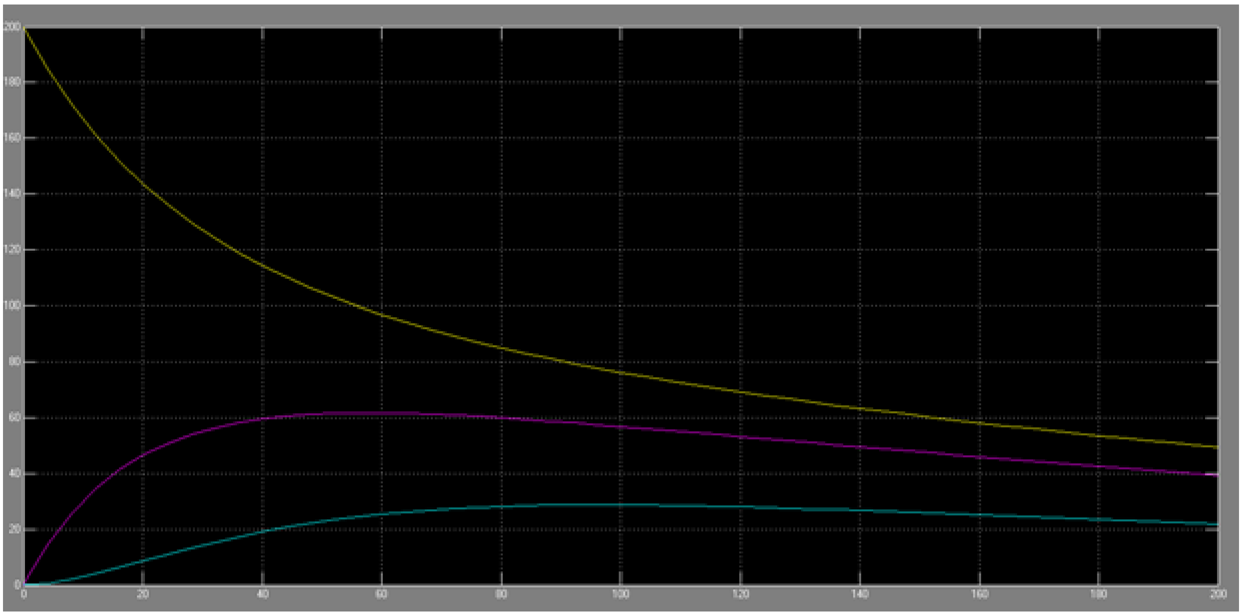

Figure A5.Comparison scopes