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RESEARCH ARTICLE

Perturbation Unsteady Flows of 1-D Fluid

***Muhammad Raheel Mohyuddin[1,2], Samra[2], Syed Mohammad Rizwan[1]**
[1]Department of Mathematics, Caledonian College of Engineering, Oman.
[2]NCBAE, Gujrat, Pakistan.

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ABSTRACT

This paper studies about the grade-III fluid having unidirectional and unsteady flow, possessing acceleration. Non-linear partial differential equation, having the slip condition, is solved using perturbation method & similarity transform. The Newtonian fluid is determined by Navier having 3-unsteady, non-linear, partial differential equations. The solution of this fluid is solved using the similarity and by applying perturbation method at zeroth and first order. The zeroth velocity (u_0) solution leads to the solution of grade-II fluid and first velocity (u_1) contribute in finding the solution of the grade-III fluid.

Keywords: Acceleration, Perturbation method, Slip, Fluid, Viscosity.

1. INTRODUCTION

The flow that guides the Newtonian fluid is the proportionality given by Newton's law of viscosity. The Newtonian fluid is determined by Navier having 3-unsteady, non-linear, partial differential equations. The solution for these partial differential equations is given in the references [1, 2, 3]. In the grade fluids (where shear stress is not proportional to rate of change) the viscosity & additional substantial factors are constant. These fluids have, in general, equations in higher order than the Newtonian equations and there is a need to have additional conditions. To remove this problem, we get the perturbation method using the similarity transform [4, 5]. The zeroth solution of the problem is solved using the exact method to get $u_0(y,t)$. This zeroth solution is then used in first order velocity $u_1(y,t)$. With this we get perturbation solution at zeroth & first level [4, 5, 6, 7].

This study gives the unsteady partial differential equations for Newtonian & viscous fluid having the slip conditions & acceleration $y \geq 0$ [7]. We have given the solution in terms of perturbation with similarities. The solutions of these perturbation solutions gives the solution of PDE.

2. FORMULATION

Grade 3 fluid has the form [3]

$$\Upsilon = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2(A_1A_2 + A_2A_1) + \beta_3(trA_1^2)A_1 \qquad (2.1)$$

where p is the pressure, **I** is unit tensor, μ is the co-efficient of viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ the materials of fluids and $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}$ are the first three Rivlin-Ericksen [3] tensors given by

$$A1 = \nabla V + (\nabla V)^T \qquad (2.2)$$

$$A_2 = \frac{dA_1}{dt} + A_1(\nabla V) + (\nabla V)^T A_1 \qquad (2.3)$$

$$A_3 = \frac{dA_2}{dt} + A_2(\nabla V) + (\nabla V)^T A_2 \qquad (2.4)$$

$$\nabla V = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix} \qquad (2.5)$$

where **V** denotes the velocity vector & d/dt is the material time differentiation.
The velocity is of the form [4]

*Corresponding author. Tel.: +96899852897
Email address: mraheel.mohyuddin@caledonian.edu.om (M.R.Mohyuddin)
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$$\mathbf{V} = [\mathrm{u(y,t)}, 0, 0] \quad (2.6)$$

where $u(y,t)$ is the velocity in x-direction.

Using (2.6) in (2.1) to (2.5) gives the following equation [3]

$$b\left(\frac{\partial u}{\partial t}\right) = \nu \frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial^3 u}{\partial t \partial y^2} + \Lambda \frac{\partial^4 u}{\partial t^2 \partial y^2} + \varepsilon \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^3 \quad (2.7)$$

where,

$$\nu = \frac{\mu}{\rho}, \alpha = \frac{\alpha_1}{\rho}, \Lambda = \frac{\beta_1}{\rho}, \varepsilon = \frac{6(\beta_2 + \beta_3)}{\rho}$$

Equation (2.7) is the partial differential equation for unidirectional flow of grade-III fluid [8, 9].

3. BOUNDARY CONDITIONS

The solution of equation (2.7) requires boundary conditions. These are given by

$$u(y,t) - \epsilon \left[\frac{\partial u}{\partial y} + \frac{\alpha}{v}\frac{\partial^2 u}{\partial y \partial t} + \frac{\beta}{v}\left(\frac{\partial u}{\partial y}\right)^3 + \frac{\Lambda}{v}\frac{\partial^3 u}{\partial y^2 \partial t}\right] = Ue^{(A+ia)t}, y = 0 \quad (3.1)$$

$$u(y,0) = 0 \quad (3.2)$$

$$u(\infty,t) = 0 \quad (3.3)$$

where

A is the acceleration factor & ϵ is the slip.

We use the perturbation in the velocity to get the linear forms of equation (2.7) and (3.1), and then apply similarity to get the solution.

4. SOLUTION

The solution of equation (2.7) is given in terms of perturbation in ε:

$$u(y,t;\varepsilon) = u_0(y,t) + \varepsilon u_1(y,t) \quad (4.1)$$

Using equation (4.1) in (2.7) to (3.3) having the similar powers of ε, we get the following equations:

4.1. Zeroth form

$$\frac{\partial u_0}{\partial t} = v \frac{\partial^2 u_0}{\partial y^2} + \alpha \frac{\partial^3 u_0}{\partial y^2 \partial t} + \Lambda \frac{\partial^4 u_0}{\partial y^2 \partial t^2} \quad (4.2)$$

$$u_0 - \epsilon \left[\frac{\partial u_0}{\partial y} + \frac{\alpha}{v}\frac{\partial^2 u_0}{\partial y \partial t} + \frac{\Lambda}{v}\frac{\partial^3 u_0}{\partial y^2 \partial t}\right] = Ue^{(A+ia)t} \quad, y = 0 \quad (4.3)$$

$$u_0(\infty, t) = 0 \quad (4.4)$$

4.2. First form

The details are given by means of equations from (4.5) to (4.8).

$$\frac{\partial u_1}{\partial t} = v \frac{\partial^2 u_1}{\partial y^2} + \alpha \frac{\partial^3 u_1}{\partial y^2 \partial t} + \Lambda \frac{\partial^4 u_1}{\partial y^2 \partial t^2} + \frac{\partial}{\partial y}\left(\frac{\partial u_0}{\partial y}\right)^3 \quad (4.5)$$

$$u1 - \epsilon \left[\frac{\partial u_1}{\partial y} + \frac{\alpha}{v}\frac{\partial^2 u_1}{\partial y \partial t} + \frac{1}{v}\left(\frac{\partial u_0}{\partial y}\right)^3 + \frac{\Lambda}{v}\frac{\partial^3 u_1}{\partial y^2 \partial t}\right] = 0, y = 0 \quad (4.6)$$

$$u_1(\infty, t) = 0 \quad (4.7)$$

The solution of equation (4.2) subject to (4.3) and (4.4) is of the form

$$u_0(y,t) = f(y)e^{(A+ia)t} \quad (4.8)$$

Using [18] in equation (4.2), (4.3) and (4.4), we have

$$f''(y) - A_1 f(y) = 0 \quad (4.9)$$

$$f(\infty) = 0 \quad (4.10)$$

$$f(0) - \epsilon \left[\left(1 + \frac{\alpha}{v}(A + ia)\right) f'(0) + \frac{\Lambda}{v}(A + ia) f''(0)\right] = U \quad (4.11)$$

Solving (4.9) subject to (4.10) & (4.11) gives (4.12)

$$f(y) = \frac{U}{1-\epsilon\left[\left(1+\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_1}\right)+\frac{\Lambda}{v}(A+ia)A_1\right]} e^{-\sqrt{A_1}y} \quad (4.12)$$

The unsteady velocity profile has the form

$$u_0(y,t) = \frac{U}{1-\epsilon\left[\left(1+\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_1}\right)+\frac{\Lambda}{v}(A+ia)A_1\right]} e^{-\sqrt{A_1}y} e^{(A+ia)t} \quad (4.13)$$

Differentiating equation (4.13) in (4.5) & (4.6), we get

$$\frac{\partial u_1}{\partial t} = v\frac{\partial^2 u_1}{\partial y^2} + \alpha\frac{\partial^3 u_1}{\partial y^2 \partial t} + \Lambda\frac{\partial^4 u_1}{\partial y^2 \partial t^2} + 3{B_1}^3 U^3 {A_1}^2 e^{-3\sqrt{A_1}y} e^{3(A+ia)t} \quad (4.14)$$

$$u_1 - \epsilon\left[\frac{\partial u_1}{\partial y} + \frac{\alpha}{v}\frac{\partial^2 u_1}{\partial y \partial t} + \frac{1}{v}\left(A_4\sqrt{A_1}\right)^3 e^{-\sqrt{A_1}y} e^{3(A+ia)t} + \frac{\Lambda}{v}\frac{\partial^3 u_1}{\partial y^2 \partial t}\right] = 0, y = 0 \quad (4.15)$$

where

$$A_4 = \frac{U}{1-\epsilon\left[\left(1+\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_1}\right)+\frac{\Lambda}{v}(A+ia)A_1\right]}$$

The solution of equation (4.14) subject to (4.15) and (4.7) is given as

$$u_1(y,t) = F(y)e^{3(A+ia)t} \quad (4.16)$$

Using (4.16) in equation (4.14), (4.15) and (4.7), we have

$$F''(y) - A_2F(y) = -A_3 e^{-\sqrt{A_1}y} \quad (4.17)$$

$$F(0) - \epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)F'(0) + 3\frac{\Lambda}{v}(A+ia)F''(0) + \frac{1}{v}\left(U^3{A_4}^3\left(-\sqrt{A_4}\right)^3\right)\right] = 0 \quad (4.18)$$

$$F(\infty) = 0 \quad (4.19)$$

where

$$A_2 = \frac{3(A+ia)}{(v+\alpha 3(A+ia) - \Lambda 9a^2)}$$

$$A_3 = \frac{3U^3{A_4}^3{A_1}^2}{(v+\alpha 3(A+ia) - \Lambda 9a^2)}$$

The solution of (4.17) subject to (4.18) & (4.19) is given by (4.20)

$$F(y) = \frac{a_2 + \epsilon\left[3a_2\left(\sqrt{A_1}\left(1+3\frac{\alpha}{v}(A+ia)\right) - 9A_1\frac{\Lambda}{v}(A+ia)\right) + \frac{1}{v}\left(U^3{A_4}^3\left(\sqrt{A_1}\right)^3\right)\right]}{\left\{1-\epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_2}\right)+3\frac{\Lambda}{v}(A+ia)A_2\right]\right\}} e^{-\sqrt{A_2}y}) - \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}e^{-3\sqrt{A_1}y}\right)\frac{1}{9A_1 - A_2} \quad (4.20)$$

where

$$a_2 = \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}\right)\frac{1}{9A_1 - A_2} \quad (4.21)$$

$$u_1(y,t) = \left(\frac{a_2 + \epsilon\left[3a_2\left(\sqrt{A_1}\left(1+3\frac{\alpha}{v}(A+ia)\right) - 9A_1\frac{\Lambda}{v}(A+ia)\right) + \frac{1}{v}\left(U^3{A_4}^3\left(\sqrt{A_1}\right)^3\right)\right]}{\left\{1-\epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_2}\right)+3\frac{\Lambda}{v}(A+ia)A_2\right]\right\}} e^{-\sqrt{A_2}y} - \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}e^{-3\sqrt{A_1}y}\right)\frac{1}{9A_1 - A_2}\right) e^{3(A+ia)t} \quad (4.22)$$

The usage of solutions given in (4.13) and (4.21) in (4.22), lead to (4.23)

$$u(y,t;\varepsilon) = UAe^{-\sqrt{A_1}y}e^{(A+ia)t} + \varepsilon\left[\left(\frac{a_2 + \epsilon\left[3a_2\left(\sqrt{A_1}\left(1+3\frac{\alpha}{v}(A+ia)\right) - 9A_1\frac{\Lambda}{v}(A+ia)\right) + \frac{1}{v}\left(U^3{A_4}^3\left(\sqrt{A_1}\right)^3\right)\right]}{\left\{1-\epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_2}\right)+3\frac{\Lambda}{v}(A+ia)A_2\right]\right\}} e^{-\sqrt{A_2}y} - \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}e^{-3\sqrt{A_1}y}\right)\frac{1}{9A_1 - A_2}\right) e^{3(A+ia)t}\right] \quad (4.23)$$

5. CONCLUSION

We have given the solution of grade-III fluid having the slip condition & the acceleration of velocity at $y = 0$ [7]. The solution of this fluid is solved using the similarity & applying perturbation method at zeroth & first order. The zeroth velocity (u_0) solution leads to the solution of grade-II fluid & first velocity (u_1) contribute in the solution of the grade-III fluid.

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