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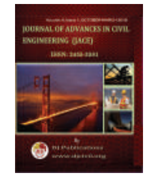

RESEARCH ARTICLE

Application of New Iterative Method and Adomian Decomposition Method to Hamel's Flow Problem

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ABSTRACT

In this work, we solve the Hamel's Problem using New Iterative Method (NIM) and Adomian Decomposition Method (ADM). This is a third order nonlinear boundary value problem. This problem arises when an incompressible fluid flows outward in a trough between two nonparallel plane walls. Application of these methods to this problem gives excellent numerical results with only three iterations. Moreover, accompanying residuals with these methods is an additional plus point which make these methods self-sufficient.
Keywords: Hamel's problem, Nonlinear boundary value problem, NIM, ADM, Incompressible fluid method.

1. INTRODUCTION

Higher order initial and boundary value problems have been investigated by many authors due to their physical importance and the potential for applications in hydrodynamic and hydro magnetic stability [1, 2]. Exact solutions for higher order differential equations are rare in literature. Researchers developed many methods to address this issue and to obtain approximate solutions of high accuracy. The most common techniques for obtaining a series form of the approximate solution, are; Adomian Decomposition Method (ADM) [3-5], variational iteration [6-8], homotopy perturbation [9-11], homotopy analysis [12], Differential Transform Method (DTM) [13] and optimal homotopy asymptotic method [14, 15].

A New Iterative Method (NIM) was introduced by Daftardar-Gejji and Jafari. It is an efficient technique to solve nonlinear functional equations [16].

Non-perturbation methods like ADM, NIM, DTM etc have certain advantages over routine numerical methods. Numerical methods use discretization which gives rise to rounding off errors causing loss of accuracy, and require large computer memory and time. On the other hand, the above mentioned methods yield analytical solutions and have certain advantages over standard numerical methods. These methods do not involve discretization of the variables and hence are free from rounding off errors and do not require large computer memory or time. In certain cases the obtained series solutions lead to the closed form solutions.

2. APPLYING NIM AND ADM

In this section we are not presenting general procedures of NIM and ADM. These methods have been extensively used in literature. We targeted these methods for the following boundary value problem [17]:

$$\frac{d^3y}{dx^3}+(2y+4)\frac{dy}{dx}=0,$$

$$y(0)=0, \qquad y'(0.087)=0, \qquad y''(0)=-0.10.$$

Let L $=\dfrac{d^3}{dx^3}$, then y $=-L^{-1}\left\{(2y+4)\dfrac{dy}{dx}\right\}$, where $L^{-1}=-\int_0^x\int_0^x\int_0^x N dx$ and, $N=(2y+4)\dfrac{dy}{dx}$.

To solve this problem, we set an algo according to the NIM and ADM. We seek the solution in the form

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$$y = y_0 + \sum_{k-1}^{\infty} y_k \tag{1}$$

where y_0 is the solution of $Ly_0 = 0$, $y_0(0) = 0$, $y_0'(0.087) = 0$, $y_0''(0) = -0.10$, and y_i is obtained from

$$y_i = -\int_0^x \int_0^x \int_0^x N_{i-1} dx$$

At this stage, we decompose $N = \sum N_j$.

Now as,

$$N = \left(2\sum_{i=0}^{\infty} y_i + 4\right) \sum_{i=0}^{\infty} y_i = \left(2(y_0 + y_1 + y_2 + \cdots) + 4\right)(y_0' + y_1' + y_2' + \cdots),$$

so, in accordance to NIM & ADM, the following decomposition takes place as shown in table 1.

Table 1.Decomposition of NIM and ADM

NIM	ADM
$N_0 = 2y_0y_0' + 4y_0'$, $N_1 = 2y_0'y_1 + 2y_1'y_0 + 2y_1'y_1 + 4y_1'$, $N_2 = 2y_0'y_2 + 2y_1'y_2 + 2y_2'y_0 + 2y_2'y_1 + 2y_2'y_2 + 4y_2'$ $\cdots$	$N_0 = 2y_0y_0' + 4y_0'$, $N_1 = 2y_0'y_1 + 2y_1'y_0 + 4y_1'$, $N_2 = 2y_0'y_2 + 2y_2'y_0 + 2y_1'y_1 + 4y_2'$ $\cdots$

The components of the solution (1) are now given by

$$y_1 = -\int_0^x \int_0^x \int_0^x N_0 dt, \quad y_2 = -\int_0^x \int_0^x \int_0^x N_1 dt, \quad y_3 = -\int_0^x \int_0^x \int_0^x N_2 dt, ...$$

Truncating the series solution (1), approximate solution is given by

$$\tilde{y} = y_0 + \sum_{k=1}^{n} y_k \tag{2}$$

Validity of the solution (2) can be confirmed by calculating residual at different mesh points of the domain. For NIM solution, the residual is

$$R_{NIM} = \frac{d^3}{dx^3}(\tilde{y}_{NIM}) + (2\tilde{y}_{NIM} + 4)\frac{d}{dx}(\tilde{y}_{NIM})$$

For ADM solution, the residual is

$$R_{ADM} = \frac{d^3}{dx^3}(\tilde{y}_{ADM}) + (2\tilde{y}_{ADM} + 4)\frac{d}{dx}(\tilde{y}_{ADM})$$

These residuals do not give the true error but they do indicate the validity of solution.
Now proceeding towards the solution we obtain, $\tilde{y} = y_0 + y_1 + y_2 + y_3 + O(x^{14})$.

$$\begin{aligned}\tilde{y}_{NIM} = &\ 0.0087x - 0.05x^2 - 0.0058x^3 + 0.0166604x^4 + 0.0012035x^5 - 0.00230135x^6 \\ &-0.000135331x^7 + 0.000193158x^8 + 6.2927\times10^{-6}x^9 - 7.0407\times10^{-6}x^{10} \\ &-7.3593\times10^{-7}x^{11} + 6.9008\times10^{-7}x^{12} + 4.6116\times10^{-8}x^{13} + O(x^{14})\end{aligned}$$

$$\begin{aligned}\tilde{y}_{ADM} = &\ 0.0087x - 0.05x^2 - 0.0058x^3 + 0.0166604x^4 + 0.0012035x^5 - 0.00230135x^6 \\ &-0.000135331x^7 + 0.000193158x^8 + 6.2927\times10^{-6}x^9 - 7.2169\times10^{-6}x^{10}\end{aligned}$$

$$-5.3134\times10^{-8}x^{11}+5.0854\times10^{-8}x^{12}+1.0844\times10^{-10}x^{13}+O(x^{14})$$

The comparison of numerical results is shown in table 2.

Table 2.Comparison of the numerical results

x	$\tilde{y}_{NIM}$	$\tilde{y}_{ADM}$	R_{NIM}	R_{ADM}
0.00	0.000	0.000	0.000	0.000
0.01	0.000081994	0.000081994	-3.0496 $\times10^{-15}$	-3.0496 $\times10^{-15}$
0.02	0.000153956	0.000153956	-1.9151 $\times10^{-13}$	-1.9167 $\times10^{-13}$
0.03	0.000215857	0.000215857	-2.1446 $\times10^{-12}$	-2.1469 $\times10^{-12}$
0.04	0.000267672	0.000267672	-1.1842 $\times10^{-11}$	-1.1859 $\times10^{-11}$
0.05	0.000309379	0.000309379	-4.4384 $\times10^{-11}$	-4.4459 $\times10^{-11}$
0.06	0.000340964	0.000340964	-1.3017 $\times10^{-10}$	-1.3042 $\times10^{-10}$
0.07	0.000362412	0.000362412	-3.2226 $\times10^{-10}$	-3.2296 $\times10^{-10}$
0.08	0.000373716	0.000373716	-7.0476 $\times10^{-10}$	-7.0440 $\times10^{-10}$

In figure 1, residual of the NIM is represented by the red solid curve and residual of the ADM is represented by the dotted curve.

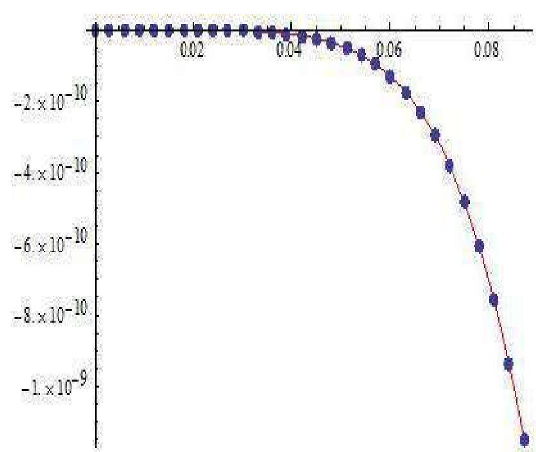

Figure 1.Red solid curve - NIM residual & dotted curve - ADM residual

3. CONCLUSION

We used NIM and ADM for the solution of Hamel's problem. Both the methods are easily applicable to higher order initial or boundary value problems and produce excellent results without any assumption and restrictions. Validity of the solutions was confirmed by calculating residuals at the mesh points. Both the methods are in complete agreement with each other.

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