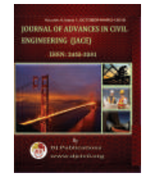

Journal of Advances in Civil Engineering, Vol. 4(1) 2018, pp. 19-23

RESEARCH ARTICLE

Plane Harmonic Elastic Waves in Homogeneous and Non-Homogeneous Isotropic Material

*A. Rehman[1], Iram Khaliq[2]

[1,2]Department of Mathematics, National College of Business Administration & Economics, Gujrat Campus, Pakistan.

Received- 4 January 2017, Revised- 26 August 2017, Accepted- 19 December 2017, Published- 29 December 2017

ABSTRACT

This article deals with the speed of two plane elastic waves namely, longitudinal and transverse waves which are propagating in homogeneous and non-homogeneous isotropic material. Inorder to calculate the speed of waves, four specimens viz, aluminium, gold, platinum and silica of homogeneous and non-homogeneous isotropic materials are used and then the speed with different velocity for the four specimen is finally compared.

Keywords: Homogeneous, Non-homogeneous, Isotropic, Waves, Material.

---

1. INTRODUCTION

Many researchers found the speed of plane elastic waves in isotropic and anisotropic materials [1]. For example, see [2, 3, 4]. Most of the investigators used the homogeneous materials, while some of the other authors used inhomogeneous mediums [4, 5, 6]. We used both of the kinds, homogeneous and non-homogeneous isotropic materials and compared the speed of the waves in these materials.

In this paper, we used the infinitessimal strain theory to study the harmonic wave motion in compressible homogeneous and non-homogeneous isotropic mediums. In persuance of the author [7], we assumed that the non-homogeneity of the elastic material is such that it grows and decays slowly, depending upon the space variable according to which it varies. Therefore, we supposed that any elastic compliance (in non-homogeneous medium), say, A° is given by [7].

$$A^\circ = Aexp[vx_1]$$

where v may be considered as a growth parameter where it is positive and decay parameter where it is negative. A is the value of A$^\circ$ when $v = 0$ (homogeneous medium).

First of all, we shall study the wave motion in homogeneous isotropic and then in non-homogeneous isotropic mediums. At the end, we shall derive the speed of these waves in both the materials, homogeneous and non-homogeneous isotopic and compare both of these velocities.

2. FORMULATIONS OF THE PROBLEM

The constitutive equations for homogenous isotropic material are [1].

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2} \end{bmatrix} \begin{bmatrix} \in_{11} \\ \in_{22} \\ \in_{33} \\ 2\in_{23} \\ 2\in_{13} \\ 2\in_{12} \end{bmatrix} \tag{1}$$

Using the relations $\in_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ (e.g see [2]) and the above equations (1) we obtain

$$\sigma_{11} = c_{11}u_{1,1} + c_{12}u_{2,2} + c_{12}u_{3,3}$$

*Corresponding author. Tel.: +923006251775
Email address: rehmanmath@yahoo.co.uk (A.Rahman)
Double blind peer review under responsibility of DJ Publications
http://dx.doi.org/10.18831/djcivil.org/2018011005

$$\begin{aligned}\sigma_{22} &= c_{12}u_{1,1}+c_{11}u_{2,2}+c_{12}u_{3,3}\\ \sigma_{33} &= c_{12}u_{1,1}+c_{12}u_{2,2}+c_{11}u_{3,3}\\ \sigma_{23} &= \frac{c_{11}-c_{12}}{2}(u_{2,3}+u_{3,2})\\ \sigma_{13} &= \frac{c_{11}-c_{12}}{2}(u_{1,3}+u_{3,1})\\ \sigma_{12} &= \frac{c_{11}-c_{12}}{2}(u_{1,2}+u_{2,1})\end{aligned} \tag{2}$$

The equations of motion are [1] (ignoring the body forces)

$$\sigma_{ij,j} = \rho\ddot{u}_i$$

These equations of motion may be written as

$$\begin{aligned}\sigma_{11,1}+\sigma_{12,2}+\sigma_{13,3} &= \rho\ddot{u}_1\\ \sigma_{21,1}+\sigma_{22,2}+\sigma_{23,3} &= \rho\ddot{u}_2\\ \sigma_{31,1}+\sigma_{32,2}+\sigma_{33,3} &= \rho\ddot{u}_3\end{aligned} \tag{3}$$

3. SPEED OF PLANE HARMONIC ELASTIC WAVES IN HOMOGENEOUS ISOTROPIC MATERIAL

A convenient representation for a displacement vector for plane harmonic waves is given by [2].

$$\overrightarrow{u} = Aexp\left[ik(\overrightarrow{x}\cdot\overrightarrow{n}-ct)\right]\overrightarrow{p}$$

where,

$$\begin{aligned}\overrightarrow{p} &= \text{a unit polarization vector}\\ \overrightarrow{n} &= \text{a unit propagation vector}\\ k &= \text{wave number and c} = \text{velocity of wave at time t}\\ A &= \text{amplitude of the wave}\end{aligned} \tag{4}$$

In view of (2,3) leads to

$$\begin{aligned}&c_{11}u_{1,11}+c_{12}u_{2,21}+c_{12}u_{3,31}+\frac{c_{11}-c_{12}}{2}(u_{1,22}+u_{2,21})+\frac{c_{11}-c_{12}}{2}(u_{1,33}+u_{3,13}) = \rho\ddot{u}_1\\ &\frac{c_{11}-c_{12}}{2}(u_{1,21}+u_{2,11})+c_{12}u_{1,12}+c_{22}u_{2,22}+c_{23}u_{3,32}+\frac{c_{11}-c_{12}}{2}(u_{2,33}+u_{3,23}) = \rho\ddot{u}_2\\ &\frac{c_{11}-c_{12}}{2}(u_{1,31}+u_{3,11})+\frac{c_{11}-c_{12}}{2}(u_{2,32}+u_{3,22})+c_{13}u_{1,13}+c_{23}u_{2,23}+c_{33}u_{3,33} = \rho\ddot{u}_3\end{aligned} \tag{5}$$

The longitudinal wave along Ox_1 whose polarization is also along Ox_1

$$\overrightarrow{n} = (1,0,0) = \overrightarrow{p}, \quad u_1 = Aexp[ik(n_1x_1-ct)], \quad u_2 = u_3 = 0$$

Therefore, equation (5a) gives the velocity of the longitudinal wave as

$$c_L = \sqrt{\frac{c_{11}}{\rho}} \tag{6}$$

For the transverse wave along Ox_1 whose polarization is parallel to Ox_2

$$\overrightarrow{n} = (1,0,0), \quad \overrightarrow{p} = (0,1,0), \quad u_2 = Aexp\left[ik(n_1x_1-ct)\right], \quad u_1 = u_3 = 0$$

Therefore, from equation (5b), the velocity of the transverse wave (whose polarization is parallel to Ox_2) is obtained as

$$c_T = \sqrt{\frac{c_{11} - c_{12}}{2\rho}} \tag{7}$$

If the polarization of the above transverse wave is parallel to Ox_3, then from (5c) again the same speed of the transverse wave is obtained. That is, speed of the transvers wave is same irrespective of the fact whether it is SV-wave or SH-wave.

4. SPEED OF PLANE HARMONIC ELASTIC WAVES IN NON-HOMOGENEOUS ISOTROPIC MATERIAL

If in (5), replace c_{ij} by $c_{ij}exp[vx_1]$ and ρ by $\rho exp[vx_1]$ respectively, then the speed of one longitudinal and one transverse waves with polarizations parallel to Ox_1, Ox_2 (or Ox_3) respectively is achieved in non-homogeneous isotropic materials as follows:

$$c_L = \sqrt{\frac{c_{11}\left[\sqrt{\frac{v^2}{k^2}+1}\right]}{\rho}}, \quad c_T = \sqrt{\frac{(c_{11}-c_{12})\left[\sqrt{\frac{v^2}{k^2}+1}\right]}{2\rho}} \tag{8}$$

5. CALCULATION OF THE SPEED OF ABOVE MENTIONED WAVES IN FOUR SPECIMEN OF HOMOGENEOUS AND NON-HOMOGENEOUS ISOTROPIC MEDIUMS

Now calculate the speed of the above mentioned waves in four specimens, aluminium, gold, platinum and silica, of homogeneous and nonhomogeneous isotropic materials. This speed is shown in the following tables 1 to 6.

Using (6-8) and the table 1, it is able to find the speed of the above mentioned two waves (one longitudinal and one transverse) and are shown in tables 2-5.

Table 1.Details about materials, stiffness and density

Material	Stiffness (10^{10} N/m^2)		Density ρ ($10^3 kg/m^3$)
	c_{11}	c_{12}	
Aluminium (Al)	10.73	6.08	2.702
Gold (Au)	19.25	16.30	19.30
Platinum (Pt)	34.70	25.10	21.40
Silica (SiO_2))	7.85	1.61	2.203

Table 2.Speed of the waves in homogeneous orthorhombic materials ($v = 0$)

Material	Speed of Longitudinal wave (m/s)	Speed of Transverse Waves (m/s)
Almuminium (Al)	6301.667	2933.428
Gold (Au)	3158.164	874.243
Platinum (Pt)	4026.785	1497.665
Silica (SiO_2)	5969.338	3766.298

Table 3.Speed of the waves in non-homogeneous orthorhombic materials $\left(0 < \frac{v^2}{k^2} < 1\right)$

Material	Value of $\frac{v^2}{k^2}$	Speed of Longitudinal Wave (m/s)	Speed of Transverse Waves (m/s)
Almuminium (Al)	0.5	6974.097	3246.445
Gold (Au)	0.5	3495.162	967.530
Platinum (Pt)	0.5	4456.470	1657.476
Silica (SiO_2)	0.5	6606.307	4168.188

Table 4.Speed of the waves in non-homogeneous orthorhombic materials $\left(\frac{v^2}{k^2} = 1\right)$

Material	Value of $\frac{v^2}{k^2}$	Speed of Longitudinal Wave (m/s)	Speed of Transverse Waves (m/s)
Almuminium (Al)	1	7493.951	3488.437
Gold (Au)	1	3755.693	1039.651
Platinum (Pt)	1	4788.658	1781.025
Silica (SiO_2)	1	7098.746	4478.887

Table 5.Speed of the waves in non-homogeneous orthorhombic materials $\left(\frac{v^2}{k^2} = 10\right)$

Material	Value of $\frac{v^2}{k^2}$	Speed of Longitudinal Wave (m/s)	Speed of Transverse Waves (m/s)
Aluminium (Al)	10	11476.302	5342.223
Gold (Au)	10	5751.501	1592.130
Platinum (Pt)	10	7333.394	2727.478
Silica (SiO_2)	10	10871.081	6859.007

Table 6.Speed of the waves in non-homogeneous orthorhombic materials $\left(\frac{v^2}{k^2} = 10^2\right)$

Material	Value of $\frac{v^2}{k^2}$	Speed of Longitudinal Wave (m/s)	Speed of Transverse Waves (m/s)
Alminium (Al)	100	19977.257	9299.420
Gold (Au)	100	3164.154	2771.484
Platinum (Pt)	100	12765.532	4759.371
Silica (SiO_2)	100	18923.725	11939.747

6. CONCLUSION

Two plane harmonic waves such as one longitudinal and one transverse waves which are propagated in homogeneous and non-homogeneous isotropic mediums. Now in this work the speed of the waves are calculated using the four specimens of both homogeneous and non-homogeneous isotropic materials. Finally on comparing the speed with different velocities of four specimens, it is found that the non-homogeneity of the material increases the speed of the waves.

REFERENCES

[1] Archit Yajnik and Rustam Ali, Numerical Solution of Integral Equations by using Discrete GHM Multi-wavelet, DJ Journal of Engineering and Applied Mathematics, Vol. 1, No. 1, 2015, pp. 17-22, http://dx.doi.org/10.18831/djmaths.org/2015011003.

[2] E.Dieulesaint and D.Royer, Elastic Waves in Solids: Applications to Signal Processing, John Wiley & Sons, Paris, 1980.

[3] J.D Achenbach, Wave Propagation in Elastic Solids, Vol. 16, Elsevier, 2012.

[4] P.Chadwick, Wave Propagation in Incompressible Transversely Isotropic Elastic Media I. Homogeneous Plane Waves, Proceedings of the Royal Irish Academy. Section A: Mathematical and Physical Sciences, Royal Irish Academy, 1993.

[5] W.T.Ang and D.L.Clements, On some Crack Problems for Inhomogeneous Elastic Materials, International Journal of Solids and Structures, Vol. 23, No. 8, 1987, pp. 1089-1104.

[6] S.Chakraborty, Effect of Nonhomogeneity on Quasi-Transverse Waves in a Nonlinear Elastic Medium, Indian Journal of Pure and Applied Mathematics, Vol. 24, 1994, pp. 467–474.

[7] D.P.Acharya and Asit Kumar Mondal, Effect of Magnetic Field on the Propagation of Quasi-Transverse Wave in a Nonhomogeneous conducting Medium under the Theory of Nonlinear Elasticity, Sadhna, Vol. 31, No. 3, 2006, pp. 199-211.