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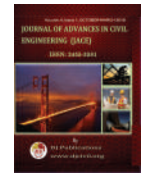

RESEARCH ARTICLE

Perturbation Unsteady Fluid Solutions of Newtonian Fluid

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ABSTRACT

In this paper, we have given the unsteady fluid solutions of Newtonian fluid. The study is given in 3-different directions, that is: (i) flow in terms of constant velocity, (ii) flow because of acceleration, (iii) flow that deforms. In these flow types, the solution is obtained by applying the Fourier transform & perturbation method in the zeroth & first velocities in the form of integrals. The graphs are given for different values of time, viscosity & Newtonian fluid showing that the velocity profile increases in terms of the values, t & α.
Keywords: 1-D fluid, Perturbation method, Fourier transform, Unsteady fluid, Newtonian fluid.

1. INTRODUCTION

The flow that gives the Newtonian fluid is the proportionality stated by Newton's Law of viscosity [1-3]. The solution for these partial differential equations is given in [4]. In the fluid of visco-elasticity the viscosity & additional substantial factors are constant, whose 1-dimensional form is given in [2].

$$\frac{\partial u}{\partial t} = v\frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial t \partial y^2}$$

These fluids have, in general, equations of higher order than the Newtonian equations & the need is to have additional conditions. In this regard, we get the perturbation method using the Fourier transform [5-7]. The zeroth solution is solved using exact method to get $u_0(y,t)$. This zeroth solution is then used in first order velocity $u_1(y,t)$. With this we get the perturbation solution at zeroth & first level [6, 7].

In this paper, three flows of visco-elastic fluids are given, namely, flow for constant velocity, flow having constant acceleration, & flow given in deformation to the fluid. [2] has solved these problems for Newtonian fluid using Fourier sine & cosine transform, whereas we have solved them using the perturbation in terms of velocity & Fourier transform. Solutions are compatible with the Newtonian solutions [2]. Graphs are presented in terms of α. The analysis is shown in equations (1) to (32).

2. CONSTANT VELOCITY

The governing equation is given in [2]

$$\frac{\partial u}{\partial t} = v\frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial t \partial y^2} \tag{1}$$

where and α are material constants.

The boundary and the initial conditions are,

$$u(y,t) = U, \quad y = 0, \quad t > 0$$
$$u(y,0) = 0, \quad y > 0$$
$$u, \quad \frac{\partial u}{\partial y} \to 0, \quad y \to \infty \tag{2}$$

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where U is the reference velocity of the plate at y=0. The solution of equation (1) subject to conditions (2) is given in perturbation [1]:

$$u(y,t) = u_0(y,t) + \alpha u_1(y,t) \tag{3}$$

Equation (1) subject to conditions (2) in view of equation (3), gives the following;

Zeroth Form:

$$\frac{\partial u_0}{\partial t} = v\frac{\partial^2 u_0}{\partial^2 y} \tag{4}$$

$$u_0(y,t) = U, \qquad y = 0$$
$$u_0(y,0) = 0, \qquad y > 0$$

$$u_0(y,t), \qquad \frac{\partial u_0(y,t)}{\partial y} \to 0, \qquad y \to \infty \tag{5}$$

First Form:

$$\frac{\partial u_1}{\partial t} = v\frac{\partial^2 u_1}{\partial^2 y} + \frac{\partial^3 u_0}{\partial t \partial^2 y} \tag{6}$$

$$u_1(y,t) = 0, \qquad y = 0$$

$$u_1(y,0) = 0, \qquad y > 0$$

$$u_1(y,t), \qquad \frac{\partial u_1(y,t)}{\partial y} \to 0, \qquad y \to \infty \tag{7}$$

The solution of equation (4) subject to condition (5) is given in Fourier sine transform.

$$u'_{os} + v\varepsilon^2 u_{os} = Uv\varepsilon$$

The solution is given by,

$$u_{os} = \frac{U}{\varepsilon}\left[1 - e^{-v\varepsilon^2 t}\right]$$

Applying the inverse sine transform, we get the solution in integral form;

$$\frac{u_0}{U} = 1 - \frac{2}{\pi}\int_0^\infty e^{-v\varepsilon^2 t}\frac{sin(y\varepsilon)}{\varepsilon}d\varepsilon \tag{8}$$

The solution given by equation (8) satisfies the conditions given in equation (5). The integral of equation (8) is

$$u_0(y,t) = ierfc\left(\frac{y}{2\sqrt{vt}}\right), \qquad U = 1 \tag{9}$$

where we have used

$$i^n erfc(x) = \int_x^\infty i^{n-1} erfc(p)dp$$

$$i^0 erfc(x) = erfc(x)$$

Using the solution (9) in (6), we get the following differential equation with initial condition,

$$u'_{1s} + v\varepsilon^2 u_{1s} = v\varepsilon^3 e^{-v\varepsilon^2 t}$$

$$u_{1s}(0) = 0$$

that has the solution

$$u_{1s} = -vt\varepsilon^3 e^{-v\varepsilon^2 t}$$

The inverse sine transform gives

$$u_1 = -\frac{2}{\pi} vt \int_0^\infty \varepsilon^3 e^{-v\varepsilon^2 t} sin(y\varepsilon) \tag{10}$$

The solution given in (10) satisfies the conditions given in (7). The integral of equation (10) forms,

$$u_1 = \frac{1}{\pi} \frac{(y^3 - 6vty)}{8(vt)^{\frac{5}{2}}} e^{-\frac{y^2}{4vt}} \tag{11}$$

Equation (3) gives the solution for the velocity profile,

$$u(y,t) = ierfc\left(\frac{y}{2\sqrt{vt}}\right) + \alpha \left[\frac{1}{\pi} \frac{(y^3 - 6vty)}{8(vt)^{\frac{5}{2}}} e^{-\frac{y^2}{4vt}}\right] \tag{12}$$

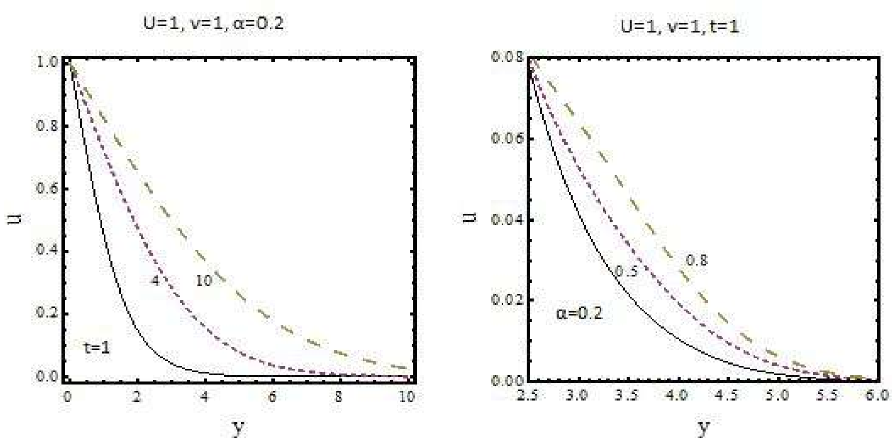

Figure 1.Velocity u(y) for value of t & α

The graphical impact of (12) for velocity u(y), with values in t & α is given in figure 1. The velocity increases from 0 to 1 for t from 1 to 10; & from 0 to 0.08 for α from 0.2 to 0.8.

3. ACCELERATION

The governing equation,

$$\frac{\partial u}{\partial t} = v\frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial^3 u}{\partial t \partial y^2}$$

having the boundary & the initial conditions are,

$$u(0,t) = At, \quad t > 0$$

$$u(y,0) = 0, \qquad y > 0$$

$$u, \qquad \frac{\partial u}{\partial y} \to 0, \qquad y \to \infty \tag{13}$$

Equation (1) subject to condition (13) in view of equation (3) gives the following:

Zeroth Form:

$$\frac{\partial u_0}{\partial t} = v\frac{\partial^2 u_0}{\partial^2 y} \tag{14}$$

$$u_0(y,t) = At, \qquad y = 0, \qquad t > 0$$

$$u_0(y,0) = 0, \qquad y > 0$$

$$u_0(y,t), \qquad \frac{\partial u_0(y,t)}{\partial y} \to 0, \qquad y \to \infty \tag{15}$$

First Form:

$$\frac{\partial u_1}{\partial t} = v\frac{\partial^2 u_1}{\partial^2 y} + \frac{\partial^3 u_0}{\partial t \partial^2 y} \tag{16}$$

$$u_1(y,t) = 0, \qquad y = 0, \qquad t > 0$$
$$u_1(y,0) = 0, \qquad y > 0$$

$$u_1(y,t), \qquad \frac{\partial u_1(y,t)}{\partial y} \to 0, \qquad y \to \infty \tag{17}$$

The solution of equation (14) subject to condition (15) is given by the Fourier sine transform;

$$u'_{os} + v\varepsilon^2 u_{os} = v\varepsilon At$$

The solution is given by,

$$u_{os} = \frac{A}{\varepsilon}\left[\frac{e^{-v\varepsilon^2 t}}{v\varepsilon^2} + t - \frac{1}{v\varepsilon^2}\right]$$

Applying the inverse sine transform, we get the solution in integral form,

$$u_0 = At - \frac{2A}{v\pi}\int_0^\infty \left[1 - e^{-v\varepsilon^2 t}\right]\frac{sin(y\varepsilon)}{\varepsilon}d\varepsilon \tag{18}$$

Solution (18) satisfies the condition given in (15). The integral of equation (18) is

$$u_0 = 4At\left(i^2 erfc\frac{y}{2\sqrt{vt}}\right) \tag{19}$$

Similarly (16) & (17) get the form

$$u'_{1s} + v\varepsilon^2 u_{1s} = A\varepsilon e^{-v\varepsilon^2 t}$$

$$u_{1s}(0) = 0$$

Integration gives

$$u_{1s} = At\varepsilon e^{-v\varepsilon^2 t}$$

Inverse Fourier transform in integral form is given by,

$$u_1 = \frac{2}{\pi}At\int_0^\infty \varepsilon e^{-v\varepsilon^2 t} sin(y\varepsilon)d\varepsilon \tag{20}$$

The integral of (20) gives,

$$u_1 = \frac{Aty}{2\sqrt{\pi}(vt)^{\frac{3}{2}}}e^{-\frac{y^2}{4vt}} \tag{21}$$

The solutions (19) & (20) in (3) implies,

$$\frac{u(y,t)}{At} = -4erfc\left(\frac{y}{2\sqrt{vt}}\right) + \frac{\alpha}{2\sqrt{\pi}}\left[\frac{y}{(vt)^{\frac{3}{2}}}e^{-\frac{y^2}{4vt}}\right] \tag{22}$$

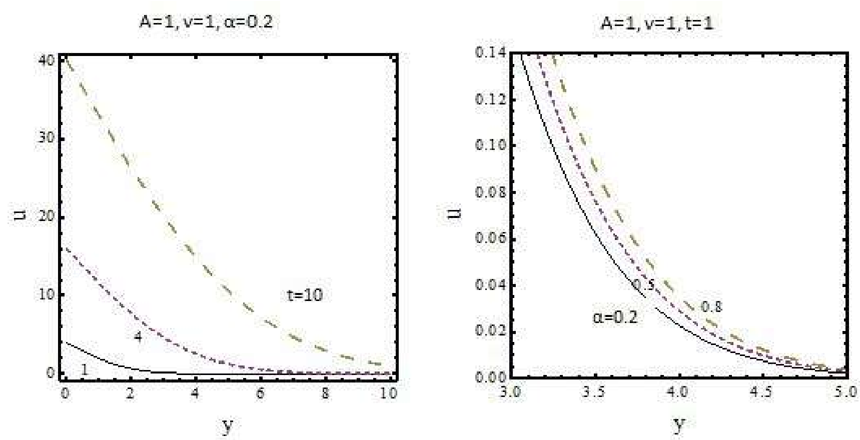

Figure 2.Velocity u(y) for value of t & α

The graphical impact of (22) for velocity u(y), with values in t & α is given in figure 2. The velocity increases from 5 to 41 for t from 1 to 10; & from 0.14 to 0.16 for α from 0.2 to 0.8.

4. CONSTANT DEFORM

Deforming of the fluid in the x-direction perpendicular to y-axis at y=0 is given by,

$$u(y,t) - \left[v\left(\frac{\partial u}{\partial y}\right) + \alpha\left(\frac{\partial^2 u}{\partial t \partial y}\right)\right] = -a, \quad y = 0 \tag{23}$$

where a is constant.

Other conditions are same as given in (2). Equation (1) subject to condition (23) in view of equation (2), gives the zeroth & first perturbation velocities.

Zeroth form:

$$\frac{\partial u_0}{\partial t} = v\frac{\partial^2 u_0}{\partial^2 y} \tag{24}$$

$$v\frac{\partial u_0}{\partial y} = -a, \qquad y = 0$$

$$u_0(y,0) = 0, \qquad y > 0$$

$$u_0(y,t), \qquad \frac{\partial u_0(y,t)}{\partial y} \to 0, \qquad y \to \infty \tag{25}$$

First form:

$$\frac{\partial u_1}{\partial t} = v\frac{\partial^2 u_1}{\partial^2 y} + \frac{\partial^3 u_0}{\partial t \partial^2 y} \tag{26}$$

$$v\frac{\partial u_1}{\partial y} + \frac{\partial^2 u_0}{\partial t \partial y} = 0, \qquad y = 0$$

$$u_1(y,0) = 0, \qquad y > 0$$

$$u_1(y,t), \qquad \frac{\partial u_1(y,t)}{\partial y} \to 0, \qquad y \to \infty \tag{27}$$

The solution of equation (24) subject to (25) in Fourier cosine transform is

$$u'_{oc} + v\varepsilon^2 u_{oc} = a$$

Solving we get,

$$u_{oc} = \frac{a}{v\varepsilon^2}\left[1 - e^{-v\varepsilon^2 t}\right]$$

Inverse transform gives

$$u_0 = \frac{2}{\pi}\int_0^\infty \frac{a}{v\varepsilon^2}\left[1 - e^{-v\varepsilon^2 t}\right] cos(y\varepsilon) d\varepsilon \tag{28}$$

Integration of (28) is,

$$u_0 = 2a\sqrt{\frac{t}{v}}\left[ierfc\left(\frac{y}{2\sqrt{vt}}\right)\right] \tag{29}$$

Using (29) in (26) & (27) & solving by Fourier cosine transform

$$u'_{1c} + v\varepsilon^2 u_{1c} = a\varepsilon^2 e^{-v\varepsilon^2 t}$$

$$u_{1c}(0) = 0$$

It is then integrated to have the form

$$u_{1c} = -at\varepsilon^2 e^{-v\varepsilon^2 t}$$

Inverse has the integral form

$$u_1 = -\frac{2}{\pi}at\int_0^\infty \varepsilon^2 e^{-v\varepsilon^2 t} cos(y\varepsilon) d\varepsilon \tag{30}$$

This is given by,

$$\frac{u_1(y,t)}{2a} = -\frac{t}{\pi}\left[\frac{e^{-\frac{y^2}{4\nu t}}y(2t-y)}{8(\sqrt{t\nu})^5}\right] \tag{31}$$

Equations (29) & (31) in (3) gives the solution for (1), (2) & (23)

$$\frac{u(y,t)}{2a} = \sqrt{\frac{t}{\nu}}\left[ierfc\left(\frac{y}{2\sqrt{\nu t}}\right)\right] - \frac{t\alpha}{\pi}\left[\frac{e^{-\frac{y^2}{4\nu t}}y(2t-y)}{8(\sqrt{t\nu})^5}\right] \tag{32}$$

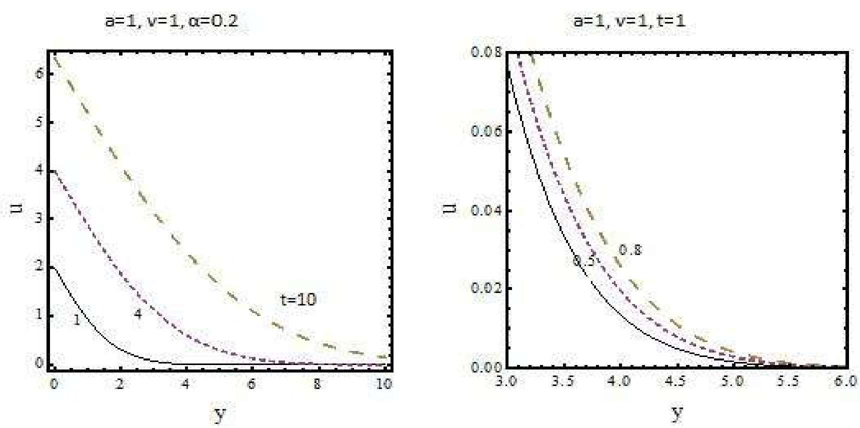

Figure 3.Velocity u(y) for t & α

The graphical impact of (32) for velocity u(y), with values in t & α is given in figure 3. The velocity increases from 2 to 6 for t from 1 to 10; & from 0.08 to 0.09 for α from 0.5 to 0.8.

5. CONCLUSION

In this paper, we have given the Newtonian fluids having 3-different flows, that is, constant velocity, acceleration, & deformation. The solution of this fluid is given via the Fourier transforms & perturbation that gives the unsolved integrals [2], solved successfully, and the solutions are given for u_0 & u_1 in terms of perturbation velocity.

The zeroth velocity (u_0) solution leads to the solution of viscous fluid & first velocity (u_1) contribute to the solution of the Newtonian fluid.

The graphs are given for different values of t & α, that shows the increase in velocity (figure 1 to figure 3).

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