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RESEARCH ARTICLE

Speed of Plane Harmonic Elastic Waves in Homogeneous and Non-Homogeneous Orthorhombic Material

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ABSTRACT

In this project we find the speed of one longitudinal and two transverse plane harmonic waves propagating in the same direction but polarizing along different directions mutually orthogonal to each other, in homogeneous and non-homogeneous orthorhombic materials. We calculated these speeds in two specimen, viz, Iodic Acid and Barium Sodium Niobate, of the said materials. From the experiments we observe that non-homogeniety of the material increases the speed of these waves.
Keywords: Non-homogeniety, Longitudinal plane harmonic waves, Transverse plane harmonic waves, Orthorhombic materials, Speed.

1. INTRODUCTION

Many researchers have studied wave motion in solids within the frame work of infinitesimal strain theory, due to which the constitutive equations [1] are taken to be linear. Therefore, the equations of motion become linear so that, it can be solved easily. For example, see [2, 3].

Many investigators have utlized the above theory to study the wave motion in isotropic and anisotropic, homogeneous and non-homogeneous, compressible and incompressible materials. For instance, see [4, 5, 6].

In this paper, we also used the above theory to study the harmonic wave motion in compressible homogeneous and non-homogeneous orthorhombic mediums. We assumed that the non-homogeneity of the elastic material is such that it grows and decays slowly, depending upon the space variable according to which it varies. Therefore, we supposed that any elastic compliance (in non-homogeneous medium), say, A^0 is given by [7].

$$A^0 = Aexp[vx_1]$$

where v_1 may be considered as a growth parameter where it is positive and decay parameter where it is negative. A is the value of A^0 when $V_1 = 0$ (homogeneous medium).

First of all, we shall study the wave motion in homogeneous orthorhombic and then in non-homogeneous orthorhombic mediums. At the end, we shall derive the speed of these waves in both the materials, homogeneous and non-homogeneous orthorhombic and compare both the velocities.

2. BASIC EQUATIONS AND FORMULATIONS OF THE PROBLEM

The analysis is carried out in equations (1) to (10).The constitutive equations for homogenous orthorhombic material are [2],

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \in_{11} \\ \in_{22} \\ \in_{33} \\ 2\in_{23} \\ 2\in_{13} \\ 2\in_{12} \end{bmatrix} \tag{1}$$

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Using the relations [2] $\in_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i})$ and the above equations we obtain,

$$\begin{aligned}
\sigma_{11} &= c_{11}u_{1,1}+c_{12}u_{2,2}+c_{13}u_{3,3}\\
\sigma_{22} &= c_{12}u_{1,1}+c_{22}u_{2,2}+c_{23}u_{3,3}\\
\sigma_{33} &= c_{13}u_{1,1}+c_{23}u_{2,2}+c_{33}u_{3,3}\\
\sigma_{23} &= c_{44}(u_{2,3}+u_{3,2})\\
\sigma_{13} &= c_{55}(u_{1,3}+u_{3,1})\\
\sigma_{12} &= c_{66}(u_{1,2}+u_{2,1})
\end{aligned} \tag{2}$$

The equations of motion are [2],

$$\sigma_{ij,j}=\rho\ddot{u}_i$$

These equations of motion may be written as,

$$\begin{aligned}
\sigma_{11,1}+\sigma_{12,2}+\sigma_{13,3} &= \rho\ddot{u}_1\\
\sigma_{21,1}+\sigma_{22,2}+\sigma_{23,3} &= \rho\ddot{u}_2\\
\sigma_{31,1}+\sigma_{32,2}+\sigma_{33,3} &= \rho\ddot{u}_3
\end{aligned} \tag{3}$$

It is a well known fact that in an anisotropic medium three plane elastic waves can propagate in a given direction $\overrightarrow{n}$, each with its own velocity. The wave whose displacement $\overrightarrow{u}^{(1)}$ (say) is closest to $\overrightarrow{n}$ is called quasi-longitudinal. Its velocity is usually greater than that of the other two waves (called quasi-transverse waves) polarized along $\overrightarrow{u}^{(2)}$ and $\overrightarrow{u}^{(3)}$ (say). The three vectors $\overrightarrow{u}^{(1)}$, $\overrightarrow{u}^{(2)}$, $\overrightarrow{u}^{(3)}$ are mutually orthogonal. Only for special propagation directions these waves are purely longitudal or transverse. If Ox_1 is one of the diad axes of the orthorhombic system and $\overrightarrow{n}$ is along this axis, then the quasi-longitudinal wave becomes purely longitudinal having polarization along $\overrightarrow{n}$ and the quasi-transverse waves become purely transverse waves having polazirations along Ox_2 and Ox_3 [2].

3. SPEED OF PLANE HARMONIC ELASTIC WAVES IN HOMOGENEOUS ORTHORHOMBIC MATERIAL

A convenient representation for a displacement vector for plane harmonic waves is given by [8].

$$\begin{gathered}
\overrightarrow{u}=Aexp\left[ik(\overrightarrow{x}\cdot\overrightarrow{n}-ct)\right]\overrightarrow{p}, \text{where}\\
\overrightarrow{p}=\text{a unit polarization vector}\\
\overrightarrow{n}=\text{a unit propagation vector}\\
k=\text{wave number and c = velocity of wave at time t}\\
A=\text{amplitude of the wave}
\end{gathered} \tag{4}$$

In view of equations (2), equations (3) of motion become,

$$\begin{aligned}
c_{11}u_{1,11}+c_{12}u_{2,21}+c_{13}u_{3,31}+c_{66}(u_{1,22}+u_{2,21})+c_{66}(u_{1,33}+u_{3,13}) &= \rho\ddot{u}_1\\
c_{66}(u_{1,21}+u_{2,11})+c_{12}u_{1,12}+c_{22}u_{2,22}+c_{23}u_{3,32}+c_{44}(u_{2,33}+u_{3,23}) &= \rho\ddot{u}_2\\
c_{55}(u_{1,31}+u_{3,11})+c_{44}(u_{2,32}+u_{3,22})+c_{13}u_{1,13}+c_{23}u_{2,23}+c_{33}u_{3,33} &= \rho\ddot{u}_3
\end{aligned} \tag{5}$$

For the purely longitudinal wave along Ox_1 whose polarization is also along Ox_1,

$$\overrightarrow{n}=(1,0,0)=\overrightarrow{p}, \quad u_1=Aexp[ik(n_1x_1-ct)], \quad u_2=u_3=0$$

Therefore, first equation of equation (5) gives the velocity of the longitudinal wave as,

$$c_L = \sqrt{\frac{c_{11}}{\rho}} \tag{6}$$

For the purely transverse wave along Ox_1 whose polarization is along Ox_2,

$$\vec{n} = (1,0,0), \quad \vec{p} = (0,1,0), \quad u_2 = Aexp\left[ik(n_1x_1 - ct)\right], \quad u_1 = u_3 = 0$$

Therefore, from the second equation of equation (5), we obtain the velocity of the transverse wave (whose polarization is along Ox_2) as,

$$c_{T_1} = \sqrt{\frac{c_{66}}{\rho}} \tag{7}$$

For the purely transverse wave along Ox_1 whose polarization is along Ox_3

$$\vec{n} = (1,0,0), \quad \vec{p} = (0,0,1), \quad u_3 = Aexp\left[ik(n_1x_1 - ct)\right], \quad u_1 = u_2 = 0$$

Therefore, the third equation of equation (5) gives the velocity of the transverse wave (whose polarization is along Ox_3) as,

$$c_{T_2} = \sqrt{\frac{c_{55}}{\rho}} \tag{8}$$

4. SPEED OF PLANE HARMONIC ELASTIC WAVES IN NON-HOMOGENEOUS ORTHORHOMBIC MATERIAL

If in equation (2) we replace c_{ij} and ρ by $c_{ij}exp[vx_1]$ and $\rho exp[vx_1]$ respectively and proceed as above, we shall get the speed of one purely longitudinal and two purely transverse waves with polarizations along Ox_1, Ox_2, Ox_3 respectively in non- homogeneous orthorhombic materials as follows:

$$c_L = \sqrt{\frac{c_{11}\left[\sqrt{\frac{v^2}{k^2}+1}\right]}{\rho}}, \quad c_{T_1} = \sqrt{\frac{c_{66}\left[\sqrt{\frac{v^2}{k^2}+1}\right]}{\rho}}, \quad c_{T_2} = \sqrt{\frac{c_{55}\left[\sqrt{\frac{v^2}{k^2}+1}\right]}{\rho}} \tag{9}$$

5. CALCULATION OF THE SPEED OF ABOVE MENTIONED WAVES IN TWO SPECIMEN OF HOMOGENEOUS AND NON-HOMOGENEOUS ORTHORHOMBIC MEDIUMS

Now we calculate the speed of the above mentioned waves in two specimen, Iodic Acid and Barium Sodium Niobate, of homogeneous and non-homogeneous orthorhombic materials. This speed is shown in the following tables.

Using equations (6), (7), (8), (9) and the table 1 [2], we can find the speed of the above mentioned three waves and are shown in table 2, table 3, table 4, table 5.

Table 1.Details about materials, stiffness and density

Material	Stiffness ($10^{10}N/m^2$)					Density $\rho(10^3Kg/m^3)$
	c_{11}	c_{13}	c_{33}	c_{55}	c_{66}	
Iodic acid HIO_3	3.01	1.11	4.29	2.06	1.58	4.64
Barium sodium niobate $Ba_2NaNb_5O_{15}$	23.9	5.00	13.5	6.60	7.60	5.30

Table 2.Speed of the waves in homogeneous orthorhombic materials ($\nu = 0$)

Material	Speed of Longitudinal wave (m/s)	Speed of Transverse Waves (m/s)
Iodic Acid	$c_L = 2546.97$	$c_{T_1} = 1845.31$, $c_{T_2} = 2107.048$
Barium Sodium Niobate	$c_L = 2269.55$	$c_{T_1} = 4047.14$, $c_{T_2} = 3771.49$

Table 3.Speed of the waves in non-homogeneous orthorhombic materials $\left(0 < \frac{\nu^2}{k^2} < 1\right)$

Material	Value of $\frac{\nu^2}{k^2}$	Speed of Longitudinal Wave (m/s)	Speed of Transverse Waves (m/s)
Iodic Acid	0.5	$c_L = 3119.43$	$c_{T_1} = 2260.062$, $c_{T_2} = 2580.69$
Barium Sodium Niobate	0.5	$c_{T1} = 2779.65$	$c_{T_1} = 4956.76$, $c_{T_2} = 4619.17$

Table 4.Speed of the waves in non-homogeneous orthorhombic materials $\left(\frac{\nu^2}{k^2} = 1\right)$

Material	Value of $\frac{\nu^2}{k^2}$	Speed of Longitudinal Wave (m/s)	Speed of Transverse Waves (m/s)
Iodic Acid	1	$c_L = 3601.95$	$c_{T_1} = 2609.66$, $c_{T_2} = 2979.81$
Barium Sodium Niobate	1	$c_L = 3209.62$	$c_{T_1} = 5723.51$, $c_{T_2} = 5333.68$

Table 5.Speed of the waves in non-homogeneous orthorhombic materials $\left(\frac{\nu^2}{k^2} > 1\right)$

Material	Value of $\frac{\nu^2}{k^2}$	Speed of Longitudinal Wave (m/s)	Speed of Transverse Waves (m/s)
Iodic Acid	10	$c_L = 8447.36$	$c_{T_1} = 6120.21$, $c_{T_2} = 6988.30$
Barium Sodium Niobate	10	$c_L = 7527.26$	$c_{T_1} = 13422.87$, $c_{T_2} = 12508.64$

6. CONCLUSION

We have derived the speed of three plane harmonic elastic waves (one longitudinal and two transverse) propagating in the same direction but with different polarization directions mutually perpendicular to each other, in homogeneous and non-homogeneous orthorhombic mediums. We have calculated the speeds of these waves in two specimen, Iodic Acid [9] and Barium Sodium Niobate, of homogeneous and non-homogeneous orthorhombic materials and are shown in tables 2-5 by taking different cases of $\frac{\nu^2}{k^2}$. We came to know that the non-homogeneity of the material increases the speed of these waves.

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