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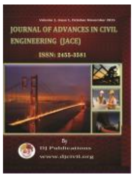

RESEARCH ARTICLE

# Perturbation Unsteady Flows of 1-D Fluid

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## ABSTRACT

This paper studies about the grade-III fluid having unidirectional and unsteady flow, possessing acceleration. Non-linear partial differential equation, having the slip condition, is solved using perturbation method & similarity transform. The Newtonian fluid is determined by Navier having 3-unsteady, non-linear, partial differential equations. The solution of this fluid is solved using the similarity and by applying perturbation method at zeroth and first order. The zeroth velocity $(u_0)$ solution leads to the solution of grade-II fluid and first velocity $(u_1)$ contribute in finding the solution of the grade-III fluid.

**Keywords:** Acceleration, Perturbation method, Slip, Fluid, Viscosity.

## 1. INTRODUCTION

The flow that guides the Newtonian fluid is the proportionality given by Newton's law of viscosity. The Newtonian fluid is determined by Navier having 3-unsteady, non-linear, partial differential equations. The solution for these partial differential equations is given in the references [1, 2, 3]. In the grade fluids (where shear stress is not proportional to rate of change) the viscosity & additional substantial factors are constant. These fluids have, in general, equations in higher order than the Newtonian equations and there is a need to have additional conditions. To remove this problem, we get the perturbation method using the similarity transform [4, 5]. The zeroth solution of the problem is solved using the exact method to get $u_0(y,t)$. This zeroth solution is then used in first order velocity $u_1(y,t)$. With this we get perturbation solution at zeroth & first level [4, 5, 6, 7].

This study gives the unsteady partial differential equations for Newtonian & viscous fluid having the slip conditions & acceleration $y \geq 0$ [7]. We have given the solution in terms of perturbation with similarities. The solutions of these perturbation solutions gives the solution of PDE.

## 2. FORMULATION

Grade 3 fluid has the form [3]

$$\Upsilon = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2(A_1A_2 + A_2A_1) + \beta_3(trA_1^2)A_1 \quad (2.1)$$

where p is the pressure, **I** is unit tensor, μ is the co-efficient of viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ the materials of fluids and $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}$ are the first three Rivlin-Ericksen [3] tensors given by

$$A1 = \nabla V + (\nabla V)^T \quad (2.2)$$

$$A_2 = \frac{dA_1}{dt} + A_1(\nabla V) + (\nabla V)^T A_1 \quad (2.3)$$

$$A_3 = \frac{dA_2}{dt} + A_2(\nabla V) + (\nabla V)^T A_2 \quad (2.4)$$

$$\nabla V = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix} \quad (2.5)$$

where **V** denotes the velocity vector & $d/_{dt}$ is the material time differentiation.
The velocity is of the form [4]

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$$\mathbf{V} = [\mathrm{u}(\mathrm{y},\mathrm{t}), 0, 0] \tag{2.6}$$

where $u(y,t)$ is the velocity in $x$-direction.

Using (2.6) in (2.1) to (2.5) gives the following equation [3]

$$b\left(\frac{\partial u}{\partial t}\right) = \nu\frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial t \partial y^2} + \Lambda\frac{\partial^4 u}{\partial t^2 \partial y^2} + \varepsilon\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^3 \tag{2.7}$$

where,

$$\nu = \frac{\mu}{\rho}, \alpha = \frac{\alpha_1}{\rho}, \Lambda = \frac{\beta_1}{\rho}, \varepsilon = \frac{6(\beta_2+\beta_3)}{\rho}$$

Equation (2.7) is the partial differential equation for unidirectional flow of grade-III fluid [8, 9].

## 3. BOUNDARY CONDITIONS

The solution of equation (2.7) requires boundary conditions. These are given by

$$u(y,t) - \epsilon\left[\frac{\partial u}{\partial y} + \frac{\alpha}{\nu}\frac{\partial^2 u}{\partial y \partial t} + \frac{\beta}{\nu}\left(\frac{\partial u}{\partial y}\right)^3 + \frac{\Lambda}{\nu}\frac{\partial^3 u}{\partial y^2 \partial t}\right] = Ue^{(A+ia)t}, y = 0 \tag{3.1}$$

$$u(y,0) = 0 \tag{3.2}$$

$$u(\infty,t) = 0 \tag{3.3}$$

where
A is the acceleration factor & $\epsilon$ is the slip.

We use the perturbation in the velocity to get the linear forms of equation (2.7) and (3.1), and then apply similarity to get the solution.

## 4. SOLUTION

The solution of equation (2.7) is given in terms of perturbation in $\varepsilon$:

$$u(y,t;\varepsilon) = u_0(y,t) + \varepsilon u_1(y,t) \tag{4.1}$$

Using equation (4.1) in (2.7) to (3.3) having the similar powers of ε, we get the following equations:

### 4.1. Zeroth form

$$\frac{\partial u_0}{\partial t} = \nu\frac{\partial^2 u_0}{\partial y^2} + \alpha\frac{\partial^3 u_0}{\partial y^2 \partial t} + \Lambda\frac{\partial^4 u_0}{\partial y^2 \partial t^2} \tag{4.2}$$

$$u_0 - \epsilon\left[\frac{\partial u_0}{\partial y} + \frac{\alpha}{\nu}\frac{\partial^2 u_0}{\partial y \partial t} + \frac{\Lambda}{\nu}\frac{\partial^3 u_0}{\partial y^2 \partial t}\right] = Ue^{(A+ia)t} \quad, y = 0 \tag{4.3}$$

$$u_0(\infty,t) = 0 \tag{4.4}$$

### 4.2. First form

The details are given by means of equations from (4.5) to (4.8).

$$\frac{\partial u_1}{\partial t} = \nu\frac{\partial^2 u_1}{\partial y^2} + \alpha\frac{\partial^3 u_1}{\partial y^2 \partial t} + \Lambda\frac{\partial^4 u_1}{\partial y^2 \partial t^2} + \frac{\partial}{\partial y}\left(\frac{\partial u_0}{\partial y}\right)^3 \tag{4.5}$$

$$u1 - \epsilon\left[\frac{\partial u_1}{\partial y} + \frac{\alpha}{\nu}\frac{\partial^2 u_1}{\partial y \partial t} + \frac{1}{\nu}\left(\frac{\partial u_0}{\partial y}\right)^3 + \frac{\Lambda}{\nu}\frac{\partial^3 u_1}{\partial y^2 \partial t}\right] = 0, y = 0 \tag{4.6}$$

$$u_1(\infty,t) = 0 \tag{4.7}$$

The solution of equation (4.2) subject to (4.3) and (4.4) is of the form

$$u_0(y,t) = f(y)e^{(A+ia)t} \tag{4.8}$$

Using [18] in equation (4.2), (4.3) and (4.4), we have

$$f''(y) - A_1 f(y) = 0 \tag{4.9}$$

$$f(\infty) = 0 \tag{4.10}$$

$$f(0) - \epsilon\left[\left(1 + \frac{\alpha}{\nu}(A+ia)\right)f'(0) + \frac{\Lambda}{\nu}(A+ia)f''(0)\right] = U \tag{4.11}$$

Solving (4.9) subject to (4.10) & (4.11) gives (4.12)

$$f(y) = \frac{U}{1-\epsilon\left[\left(1+\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_1}\right)+\frac{\Lambda}{v}(A+ia)A_1\right]} e^{-\sqrt{A_1}y} \quad (4.12)$$

The unsteady velocity profile has the form

$$u_0(y,t) = \frac{U}{1-\epsilon\left[\left(1+\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_1}\right)+\frac{\Lambda}{v}(A+ia)A_1\right]} e^{-\sqrt{A_1}y} e^{(A+ia)t} \quad (4.13)$$

Differentiating equation (4.13) in (4.5) & (4.6), we get

$$\frac{\partial u_1}{\partial t} = v\frac{\partial^2 u_1}{\partial y^2} + \alpha\frac{\partial^3 u_1}{\partial y^2 \partial t} + \Lambda\frac{\partial^4 u_1}{\partial y^2 \partial t^2} + 3{B_1}^3 U^3 {A_1}^2 e^{-3\sqrt{A_1}y} e^{3(A+ia)t} \quad (4.14)$$

$$u_1 - \epsilon\left[\frac{\partial u_1}{\partial y} + \frac{\alpha}{v}\frac{\partial^2 u_1}{\partial y \partial t} + \frac{1}{v}\left(A_4\sqrt{A_1}\right)^3 e^{-\sqrt{A_1}y} e^{3(A+ia)t} + \frac{\Lambda}{v}\frac{\partial^3 u_1}{\partial y^2 \partial t}\right] = 0, y = 0 \quad (4.15)$$

where

$$A_4 = \frac{U}{1-\epsilon\left[\left(1+\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_1}\right)+\frac{\Lambda}{v}(A+ia)A_1\right]}$$

The solution of equation (4.14) subject to (4.15) and (4.7) is given as

$$u_1(y,t) = F(y)e^{3(A+ia)t} \quad (4.16)$$

Using (4.16) in equation (4.14), (4.15) and (4.7), we have

$$F''(y) - A_2F(y) = -A_3 e^{-\sqrt{A_1}y} \quad (4.17)$$

$$F(0) - \epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)F'(0) + 3\frac{\Lambda}{v}(A+ia)F''(0) + \frac{1}{v}\left(U^3{A_4}^3\left(-\sqrt{A_4}\right)^3\right)\right] = 0 \quad (4.18)$$

$$F(\infty) = 0 \quad (4.19)$$

where

$$A_2 = \frac{3(A+ia)}{(v+\alpha 3(A+ia)-\Lambda 9a^2)}$$

$$A_3 = \frac{3U^3{A_4}^3{A_1}^2}{(v+\alpha 3(A+ia)-\Lambda 9a^2)}$$

The solution of (4.17) subject to (4.18) & (4.19) is given by (4.20)

$$F(y) = \frac{a_2+\epsilon\left[3a_2\left(\sqrt{A_1}\left(1+3\frac{\alpha}{v}(A+ia)\right)-9A_1\frac{\Lambda}{v}(A+ia)\right)+\frac{1}{v}\left(U^3{A_4}^3\left(\sqrt{A_1}\right)^3\right)\right]}{\left\{1-\epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_2}\right)+3\frac{\Lambda}{v}(A+ia)A_2\right]\right\}} e^{-\sqrt{A_2}y}) - \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}e^{-3\sqrt{A_1}y}\right)\frac{1}{9A_1-A_2} \quad (4.20)$$

where

$$a_2 = \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}\right)\frac{1}{9A_1-A_2} \quad (4.21)$$

$$u_1(y,t) = \left(\frac{a_2+\epsilon\left[3a_2\left(\sqrt{A_1}\left(1+3\frac{\alpha}{v}(A+ia)\right)-9A_1\frac{\Lambda}{v}(A+ia)\right)+\frac{1}{v}\left(U^3{A_4}^3\left(\sqrt{A_1}\right)^3\right)\right]}{\left\{1-\epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_2}\right)+3\frac{\Lambda}{v}(A+ia)A_2\right]\right\}} e^{-\sqrt{A_2}y} - \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}e^{-3\sqrt{A_1}y}\right)\frac{1}{9A_1-A_2}\right)e^{3(A+ia)t} \quad (4.22)$$

The usage of solutions given in (4.13) and (4.21) in (4.22), lead to (4.23)

$$u(y,t;\varepsilon) = UAe^{-\sqrt{A_1}y}e^{(A+ia)t} + \varepsilon\left[\left(\frac{a_2+\epsilon\left[3a_2\left(\sqrt{A_1}\left(1+3\frac{\alpha}{v}(A+ia)\right)-9A_1\frac{\Lambda}{v}(A+ia)\right)+\frac{1}{v}\left(U^3{A_4}^3\left(\sqrt{A_1}\right)^3\right)\right]}{\left\{1-\epsilon\left[\left(1+3\frac{\alpha}{v}(A+ia)\right)\left(-\sqrt{A_2}\right)+3\frac{\Lambda}{v}(A+ia)A_2\right]\right\}} e^{-\sqrt{A_2}y} - \left(\frac{3U^3{A_4}^3{A_1}^2}{(v+3\alpha(A+ia)-\Lambda 9a^2)}e^{-3\sqrt{A_1}y}\right)\frac{1}{9A_1-A_2}\right)e^{3(A+ia)t}\right] \quad (4.23)$$

## 5. CONCLUSION

We have given the solution of grade-III fluid having the slip condition & the acceleration of velocity at $y = 0$ [7]. The solution of this fluid is solved using the similarity & applying perturbation method at zeroth & first order. The zeroth velocity $(u_0)$ solution leads to the solution of grade-II fluid & first velocity $(u_1)$ contribute in the solution of the grade-III fluid.

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